

Exercise 55

Use the Intermediate Value Theorem to show that there is a root of the given equation in the specified interval.

$$e^x = 3 - 2x, \quad (0, 1)$$

Solution

Bring all terms to one side.

$$e^x + 2x - 3 = 0, \quad (0, 1)$$

Let $f(x) = e^x + 2x - 3$. The exponential and polynomial functions are each continuous on their respective domains by Theorem 7. The sum or difference of these functions is also continuous by Theorem 4.

$$f(x) = 0, \quad (0, 1)$$

Find a value of x in the interval $[0, 1]$ so that $f(x)$ is negative, and find a value of x in the interval $[0, 1]$ so that $f(x)$ is positive.

$$f(0) = -2$$

$$f(1) \approx 1.72$$

$f(x)$ is continuous on the closed interval $[0, 1]$, and $N = 0$ lies between $f(0)$ and $f(1)$. By the Intermediate Value Theorem, then, there exists a number c such that $f(c) = 0$.